MAD-TH-11-11

Towards the string dual of tumbling and cascading gauge theories

Eduardo Conde,[1] Jérôme Gaillard,[2] Carlos Núñez,[3] Maurizio Piai,[3] and Alfonso V. Ramallo[1]

[1] *Departamento de Física de Partículas*
and
Instituto Galego de Física de Altas Enerxías (IGFAE),
Universidade de Santiago de Compostela
E-15782, Santiago de Compostela, Spain
[2] *Physics Department, University of Wisconsin-Madison, 1150 University Avenue*
Madison, WI 53706-1390. USA
[3] *Swansea University, School of Physical Sciences,*
Singleton Park, Swansea, Wales, UK
(Dated: March 12, 2012)

We build type IIB backgrounds that can be interpreted as the dual description of field theories in which the dynamics shows many non-trivial phenomena, generalizing the baryonic branch of the Klebanov-Strassler system. We illustrate the steps of the explicit construction with a particularly interesting example. The dual field theory exhibits the expected behavior of an $\mathcal{N} = 1$ supersymmetric gauge theory which, over different ranges of the radial direction, is undergoing a cascade of Seiberg dualities, a period of running, a cascade of Higgsings (tumbling) and finally confines with the formation of a condensate.

I. INTRODUCTION.

The dynamics of four-dimensional field theories is very rich, particularly at strong coupling. Quantum effects give rise to very non-trivial renormalization-group flows (*running* of the couplings). At strong coupling many theories *confine*, and in some conditions gauge theories can undergo spontaneous symmetry breaking (*Higgsing*), possibly in multi-scale sequences (*tumbling*) [1]. Under special circumstances (close to approximate fixed points) the running may be anomalously slow (*walking*) [2]. Finally, there are examples in which two different gauge theories (with different microscopic Lagrangians) admit the same low-energy description (*Seiberg duality*) [3]. This feature can be iterated, giving rise to what is called the *duality cascade* [4–6].

On very general grounds, it is interesting to have a (weakly coupled) dual gravity description of (strongly coupled) field theories. The methods of the AdS/CFT correspondence [7] generalize to cases where the dual field theory is not conformal [8] and exhibits one or more of the dynamical features described above. This allows to test quantitatively the properties of models for which the intuition based on perturbation theory fails to provide useful guidance. Many such non-trivial features are believed to play important roles in phenomenologically relevant models (for example, in dynamical electroweak-symmetry breaking [9–11]), and the gravity duals offer an opportunity to make quantitative predictions for measurable quantities.

The literature on the subject is very rich, see for instance [6, 12–14]. In this Letter we take a further non-trivial step. We present an algorithmic procedure that allows to construct a large class of new backgrounds, dual to $\mathcal{N} = 1$ supersymmetric gauge theories. Depending on some of the integration constants and parameters of the string configuration, the four-dimensional gauge theories exhibit one or more of the features we referred to as running, Higgsing, tumbling, confinement and duality cascade.

We produce and discuss in some detail one special example of such a construction, by providing the essential technical steps, highlighting its preeminent physical properties and by commenting on generalizations of the construction itself. For full details, we refer the reader to the vast literature on the subject and to a more extensive companion paper [15].

II. THE BACKGROUNDS

We consider the class of type IIB backgrounds that can be obtained by the KK reduction of the theory on the base of a conifold, followed by a consistent truncation. Our starting configuration contains N_c units of flux for the RR form F_3 (we call this the wrapped-D5 system), to which we add (fully back-reacted) N_f smeared D5-brane sources. There exists a procedure that, starting from this comparatively well-understood system, allows to generate a large class of backgrounds that are much more general, in which also the H_3 and F_5 forms are highly non-trivial and the dual field theory has a particularly rich multi-scale dynamics. Let us illustrate in this section how this is achieved.

A. The construction.

Consider the ansatz for a background of type IIB string theory, in which the ten-dimensional space-time

consists of four Minkowski directions x^μ, a radial direction $0 \leq \rho < +\infty$ and five internal angles $\theta, \tilde{\theta}, \varphi, \tilde{\varphi}, \psi$ parameterizing a compact manifold Σ_5 [16–20]. The full background is determined by a set of functions that are assumed to depend only on ρ, and by a set of non-trivial BPS equations that can be derived from type IIB supergravity [13, 17–19].

The whole dynamics controlling the background can be summarized by the following two equations [20–22]:

$$0 = P'' + N_f\, S' + (\, P' + N_f\, S\,) \left(\frac{P' - Q' + 2N_f\, S}{P+Q} + \frac{P' + Q' + 2N_f\, S}{P-Q} - 4\coth(2\rho) \right) = 0\ , \tag{1}$$

$$Q = \coth(2\rho) \left[\int_0^\rho dx\, \frac{2N_c - N_f\, S(x)}{\coth^2(2x)} \right], \tag{2}$$

where the primed quantities refer to derivatives with respect to ρ. The system is hence controlled by the functions $P(\rho)$, $Q(\rho)$ and $S(\rho)$. Once these functions are known, one can reconstruct in a purely algebraic way the whole type IIB background, which consists of a non-trivial metric, dilaton and RR form F_3.

While the equations for P and Q are a repackaging of the BPS equations for the original ten-dimensional system, the function $S(\rho)$ has a very different meaning: it controls the profile of the N_f (flavor) branes in the radial direction. For example, a vanishing S yields the original (unflavored) wrapped-D5 system, for which $\hat{P} = 2N_c\rho$ is a special solution [13, 17]. On the other extreme, $S = 1$ corresponds to the flavored solutions extensively discussed in the literature [18–20, 23].

Following [21, 22], we will assume that S has a non-trivial ρ-dependent profile, bounded by $0 \leq S \leq 1$. In particular, we will require that S vanishes both in the deep IR (small ρ) and in the far UV (large ρ), hence ensuring that asymptotically in the UV and in the IR the system resembles very closely the original wrapped-D5 system. The latter is the main element of novelty of the proposal in this Letter. There are important, though subtle, differences in the asymptotic expansions of backgrounds obtained for such a choice of S with respect to the case $S = 0$, as we will see.

In the case of the wrapped-D5 system (with $S = 0$), Q is exactly integrable, and the generic solution of Eq. (1) depends on two integration constants $c_\pm$, which can be read from the UV expansion of the equation, see [20]. Their meaning in terms of the field-theory duals is well understood: they correspond to the insertion of a dimension-eight operator (for c_+) and to a VEV for a dimension-six operator (for c_-), see [24].

The choice of $c_\pm$ is not completely free. In particular, there exists a minimal value for c_+, close to which the background solutions approach the special case $\hat{P} = 2N_c\rho$, which does not admit a simple interpretation in terms of a local four-dimensional dual field theory. We will hence avoid this case and c_+ will be kept explicit. As for the second integration constant c_-, the fact that it is non-vanishing is connected with the appearance in the field theory of properties that resemble those of a walking field theory [25]. Also, it seems that its presence ultimately produces a very mild singularity in the deep IR. In this Letter, we will always keep $c_- = 0$, so that the IR is non-singular.

It is known [24, 26–28] that for backgrounds for which c_+ is non-trivial, there exists an algebraic procedure (which we refer to as *rotation*) that allows to construct a new one-parameter family of solutions starting from the wrapped-D5 system. In these new 'rotated' backgrounds the warp factor $\hat{h}$ is given by

$$\hat{h} = 1 - \kappa^2 e^{2\Phi} = 1 - \kappa^2 \sqrt{\frac{2e^{4\Phi_0}\sinh(2\rho)^2}{(P^2 - Q^2)(P' + N_f S)}} \geq 0\,, \tag{3}$$

where Φ is the dilaton[1], and Φ_0 and κ are constants. We focus on cases where $P \sim c_+ e^{\frac{4}{3}\rho} + \mathcal{O}(e^{-\frac{4}{3}\rho})$ for $\rho > \bar{\rho}$, where the scale $\bar{\rho}$ is determined by c_+ itself in a non-trivial way. The dilaton approaches a constant $\Phi(\infty)$ at large ρ (as opposed to being linear [13]), hence one finds the restriction $0 \leq \kappa \leq e^{-\Phi(\infty)}$.

Summarizing, if we choose a non-zero[2] κ and perform the rotation [24, 26–28], this yields a new type IIB background in which:

- the dilaton is unchanged,
- the RR form F_3 is unchanged,
- the Einstein-frame metric takes the form
$$ds^2 = e^{\Phi/2}\Big[\hat{h}^{-1/2} dx_{1,3}^2 + \hat{h}^{1/2} ds_6^2\Big], \tag{4}$$
where ds_6^2 stands for the metric of the cone over the internal manifold Σ_5,
- the NSNS H_3 is non-trivial,

[1] Notice that a factor of 4 has been here reabsorbed in $e^{4\Phi_0}$ with respect to the notation in [24, 27], where also κ is called k_2.

[2] For $\kappa = 0$ one recovers the original unrotated solution.

- the self-dual RR field F_5 is non-trivial.

Before the rotation the background is controlled by N_c D5-branes that are encoded in the flux of F_3, and by N_f D5-branes that act as sources and have a profile in the radial direction described by $S(\rho)$. After the rotation, besides these objects there is also a flux for H_3 which, together with the five-form flux F_5, encodes background D3-branes and also a number of D3 sources induced on the N_f D5 sources by the presence of the NSNS B_2 field.

The final step of the construction consists of choosing κ. It has been observed in [24] that the effect of the rotation is very dramatic in the far UV, but only sub-leading below the scale $\bar{\rho}$ (in particular, H_3 and F_5 become very small below $\bar{\rho}$). The most interesting effect of the rotation is that it modifies the warp factor in front of the Minkowski part of the metric. In particular, by dialing $\kappa = e^{-\Phi(\infty)}$ one recovers a warp factor that approximates the one of the Klebanov-Strassler background, i.e. a background that differs from being asymptotically AdS only by a logarithmic term.

In summary, we outlined a procedure that, given a function $S(\rho)$ that vanishes in the IR and in the UV (fast enough with ρ), allows to construct a background which is free of any singular behavior. This background has a metric very similar to the one of (the baryonic branch of) the Klebanov-Strassler solution [6], but differs from it by the presence of an intermediate range of ρ where the background exhibits the presence of a distribution of D5 and D3 sources. We will provide now a concrete example, before discussing the field-theory interpretation.

B. A class of solutions.

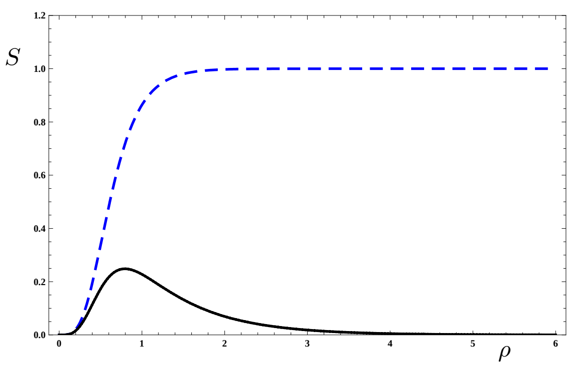

FIG. 1: The function S used in this Letter (black), compared to a particular one in [22] (dashed blue), which has $S = \tanh^4(2\rho)$.

We choose the following functional dependence for S

$$S(\rho) \; = \; \tanh^4(2\rho) e^{-4\rho/3} \, , \tag{5}$$

which we plot in Fig. 1. Before discussing the solutions, let us do some parameter counting. We keep N_c as a parameter, but set $\alpha' g_s = 1$. The end of space is fixed at $\rho_0 = 0$, which means that the function S is positive, bounded and vanishes for $\rho \to +\infty$ and for $\rho \to 0$. We set Φ_0 in such a way that $\Phi \to 1$ for $\rho \to 0$ in all the numerical solutions we study.

Finally, the whole solution depends entirely on the value of c_+, since we choose the solution for P to be linear in the IR (i.e. $P \sim h_1 \rho$), which necessarily implies $c_- = 0$ [20, 24, 27]. In the newly generated (rotated) background, the only new parameter is κ, which we fix to its maximal value $\kappa = e^{-\Phi(\infty)}$. We also set $N_f = 2N_c$, and compare in the plots, where useful, to $S = \tanh^4(2\rho)$ [22] (in the following, we refer to the latter as the *monotonic* S).

We solve numerically for P, imposing that it be linear in the IR, and plot the result for various possible solutions in Fig. 2. We find it convenient to compare to the $\hat{P} = 2N_c\rho$ solution (with $S = 0$) and to a special solution $\bar{P}$ obtained with the monotonic S. The class of solutions we find consists of the black curves in Fig. 2. The comparison shows that deep in the IR our P agrees with $\bar{P}$. However, at large ρ one recovers a behavior similar to $\hat{P}$, at least until the scale $\bar{\rho}$ at which the exponential behavior appears.

Notice that this scale $\bar{\rho}$ can be indefinitely increased, hence one might envision the case in which the asymptotics resembles that of $\hat{P}$. For this limiting case, we can approximate in the UV

$$P \simeq 2N_c\rho + 1 - 2S_\infty \, , \tag{6}$$

neglecting terms that vanish at large ρ, where the constant S_∞ is given by

$$S_\infty \; = \; \int_0^{+\infty} d\rho \tanh^2(2\rho) S(\rho) \; \simeq \; 0.29 \, . \tag{7}$$

We see that the non-trivial profile of S shifts P by a constant amount with respect to $\hat{P} = 2N_c\rho$ in the far UV.

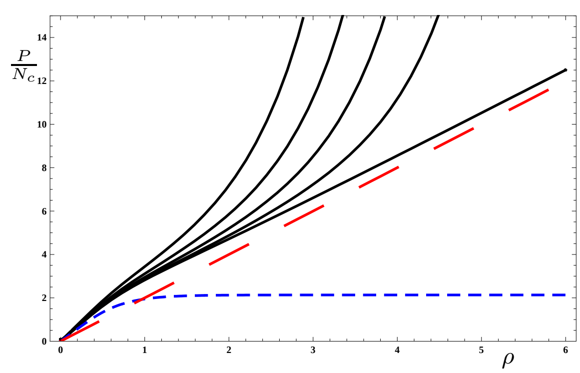

FIG. 2: The function P solving the system defined here (black), compared to one that solves the system for S as in [22] (dashed blue) and to the original $\hat{P}$ solution, obtained for $S = 0$ (long-dashed red).

More interesting for our purposes are the cases in which the UV asymptotic behavior of P is exponential, because in this case we can perform the rotation procedure. As a result, the asymptotic behavior of the solution is better understood by looking at the warp factor $\hat{h}$, which for large ρ is

$$\hat{h} = \frac{3N_c^2 e^{-\frac{8}{3}\rho}}{8c_+^2}\left(8\rho - 1 + \frac{2c_+N_f}{N_c^2} - \frac{4N_f}{N_c}S_\infty\right) + \mathcal{O}(e^{-4\rho})\,. \qquad (8)$$

This expression shows two very interesting facts: first of all, as anticipated, one obtains the familiar result that the warp factor is almost AdS, except for the term linear in ρ in parenthesis. Notice we reinstated N_f in the expansions for clarity, but one should keep in mind that in the plots we explore the case $N_f = 2N_c$.

Concluding, let us summarize what are the properties of the generic P belonging to this one-parameter family. Deep in the IR, it looks somewhat similar to $\hat{P} = 2N_c\rho$ [13], but with a slightly larger slope[3]. Over the range in which S is non-trivial, P keeps growing, but with a smaller derivative. This range gives way at intermediate values of ρ to a P which has a slope very close to $\hat{P} = 2N_c\rho$, but which is shifted upwards by a finite amount, as reflected in Eq. (6). Finally, in the far UV, P gives rise (after the rotation is performed) to a warp factor that is very close to the one of the KS case, but again with a shift in the logarithmic term. We will now go on to suggest an interpretation for these findings in field-theory terms.

III. FIELD-THEORY INTERPRETATION.

The first thing we want to study in order to provide a sensible field-theory interpretation is the central charge $c(\rho)$ of the rotated system, which turns out to be

$$c \;=\; \frac{\hat{h}^2\, e^{2\Phi}(P^2 - Q^2)(P' + N_f S)^2}{128[\partial_\rho \ln(\sqrt{\hat{h}}\, e^{2\Phi}(P^2 - Q^2)\sqrt{P' + N_f S})]^3}\,. \qquad (9)$$

The results are illustrated in Fig. 3. The central charge is positive-definite and monotonically increasing towards the UV, as expected from consistency with the c-theorem. It vanishes near the end of space, as expected because the dynamics ultimately yields confinement. Its behavior is regular, with sharp changes at the scale $\bar{\rho}$ and in the region where $S \neq 0$ (see in particular the detail of this region in Fig. 3). Notice that while qualitatively similar, the plots obtained for various choices of $P(\rho)$ are different, and agree only when $\rho \to 0$.

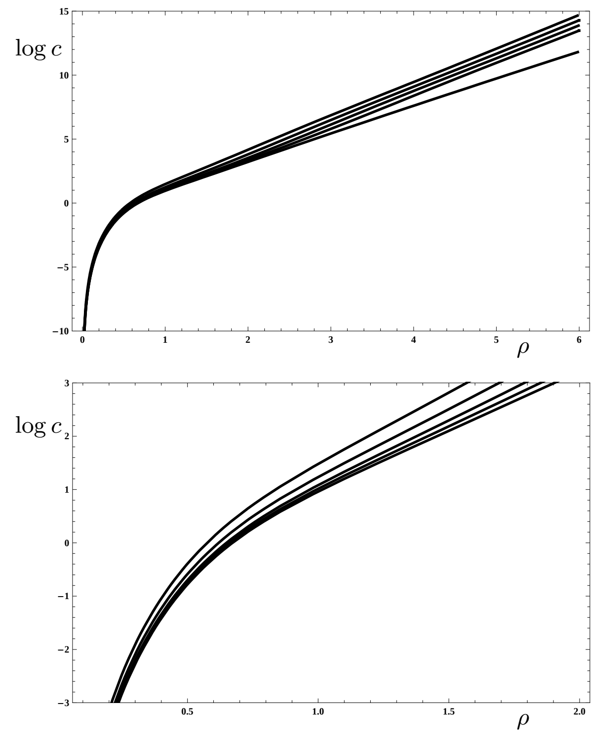

FIG. 3: The central charge c obtained with the functions P solving the system defined in the Letter, and shown as black curves in Fig. 2.

The second important quantity we want to use is the Maxwell charge of the theory, which (after the rotation) counts the total D3 charge in the background; this is made out of (flux) D3 charge plus the source D3 charge induced as an effect of the NSNS B_2 field on the world-volume of the D5 sources.

We define the number n of bulk D3-branes and n_f of source D3-branes as given by

$$\begin{aligned} n + n_f \;&\equiv\; \int_{\Sigma_5} F_5 = \int_{\Sigma_5} B_2 \wedge F_3 \;= \\ &=\; 4\pi^3 e^{\Phi(\infty)} N_c \left[\frac{N_f}{N_c} PS + 2Q\left(1 - \frac{N_f}{2N_c}S\right)\tanh(2\rho) - \frac{4}{N_c}\frac{Q^2}{\sinh(4\rho)}\right]\,. \end{aligned} \qquad (10)$$

The result of this is plotted in Fig. 4, for the same numerical solutions as in Fig. 2 and in Fig. 3. The resulting

[3] This different slope is just due to the specific functional dependence of S: one recovers the same slope as in $\hat{P} = 2N_c\rho$ by replacing the $\tanh^4(2\rho)$ factor by $\tanh^{2N}(2\rho)$ with N very large [22].

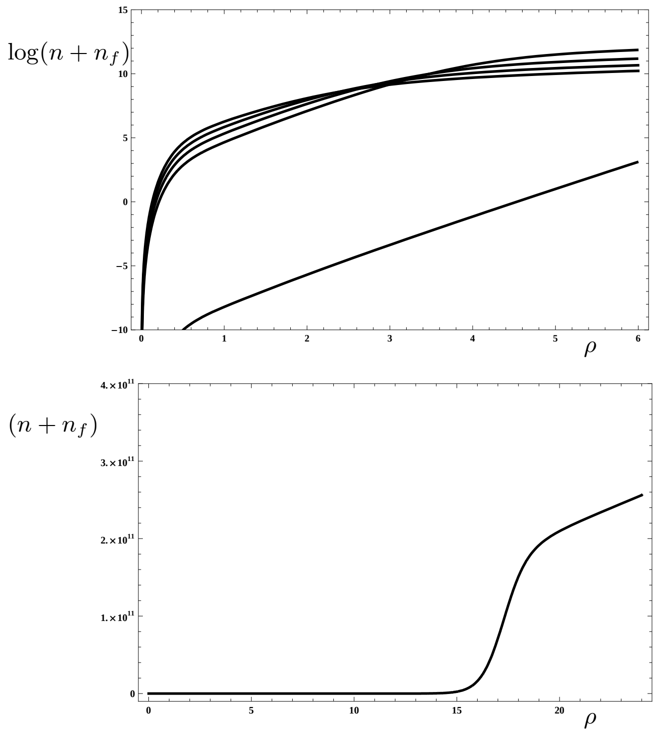

FIG. 4: The function $\log(n+n_f)$ obtained with the functions P solving the system defined in the Letter, and shown as black curves in Fig. 2. Notice that the lower curve in this plot corresponds to the one that looked linear in that figure. The lower plot shows $n+n_f$ for the numerical solution with largest $\bar{\rho}$, which clearly exhibits the sudden drop at scale $\bar{\rho}$, together with the linear dependence when $\rho > \bar{\rho}$.

functions are positive-definite, monotonically increasing, and vanish at the end of space, as required by consistency. The most visible property of $n+n_f$ is that it drops towards zero below the scale $\bar{\rho}$, which was already noticed in [24] for $N_f = 0$, and interpreted in terms of the Higgsing of the quiver into a single-site model. For $\rho > \bar{\rho}$, one sees that $n+n_f$ is linear in ρ:

$$\frac{n+n_f}{4\pi^3 N_c^2} \simeq e^{\Phi(\infty)} \left[4\rho - 2 + \frac{N_f}{N_c}\left(\frac{c_+}{N_c} - 2S_\infty\right)\right] . \quad (11)$$

We are now going to propose a possible interpretation for the dual backgrounds.

- At large scales ($\rho > \bar{\rho}$), the dual theory is a quiver with gauge group $SU(n+n_f+N_c) \times SU(n+n_f)$, where $n = kN_c$ is related to the D3 charge of the background, and n_f to the D3 sources induced by the field B_2 on the D5 sources. The theory is undergoing a cascade of Seiberg dualities of the form

$$\begin{aligned} &SU(n_f+(k+1)N_c) \times SU(n_f+kN_c) \\ \to\ &SU(n_f+(k-1)N_c) \times SU(n_f+kN_c) \\ \to\ &SU(n_f+(k-1)N_c) \times SU(n_f+(k-2)N_c) \ \to\ \cdots . \end{aligned} \quad (12)$$

- At the scale $\bar{\rho}$ the cascade stops abruptly, and the quiver theory is Higgsed down to a single-site theory because of the formation of a dimension-two condensate, which is related to the operator $\mathcal{U}$ defined in [5]. As discussed in [5, 24, 26, 29], the arbitrariness of $\bar{\rho}$ is loosely related to the modulus that controls the dimensionality of the coset. The gauge group reduces to $SU(n_f + N_c)$. For energies below the scale $\bar{\rho}$, the original wrapped-D5 system yields a good effective-field-theory description.

- In the intermediate range where $\rho < \bar{\rho}$, but where $S \simeq 0$, the theory is an $\mathcal{N} = 1$ SUSY theory with gauge group $SU(n_f+N_c)$ and massless matter fields. Besides these, as a result of the Higgsing, the spectrum of massive states deconstructs two extra dimensions, so that the theory apparently looks like a higher-dimensional field theory [29].

- In the energy range where S is non-vanishing, a cascade of Higgsings is taking place. The idea [30] is that for every source brane that is crossed when flowing down in the radial direction we Higgs the groups. This sequentially reduces further the gauge group to $SU(N_c)$, while at the same time giving mass to the matter-field content. The smooth behavior of S at large ρ yields small threshold effects, that are the reason why the $\hat{P}$ solution is not exactly reproduced, as we commented earlier.

- Very deep in the IR, close to the end of space, the theory finally resembles $\mathcal{N} = 1$ SYM with $SU(N_c)$ group in four dimensions, confines and produces a non-trivial gaugino condensate.

The dual field theory is showing, at different scales, the behavior of a field theory that undergoes a cascade of Seiberg dualities, the Higgsing of the quiver theory due to the $\mathcal{U}$ condensate, a tumbling sequence which reduces the rank of the one-site field theory, and finally confinement and gaugino condensation.

IV. OUTLOOK

There are a number of possible future research directions that can be developed starting from the results we summarized here and in a companion paper [15].

In this Letter we presented one very special class of type IIB backgrounds that exhibit some of the properties expected in field theories that have a very interesting and rich dynamics. Some of the statements we made about the interpretation in field-theory terms are mostly based on circumstantial evidence, and it would be useful to find other ways to check whether our interpretation is correct. In particular, this means that one would like to explore in a more systematic way the space of acceptable profiles for the function S, verify that the resulting backgrounds are consistent, and test whether a generalization of the

arguments we summarized here still provides a satisfactory explanation of the results. In particular, it would be interesting to see what happens when the support of S is very large, and far from the end of space of the geometry.

On the more phenomenological side, this is an interesting step in the direction of studying tumbling dynamics, which is believed to have an important role in the context of dynamical electroweak-symmetry breaking. It would be interesting to find models of this type that resemble as much as possible phenomenologically viable scenarios, and use them to compute quantities that are relevant to modern high-energy Physics.

Acknowledgments

Discussions with various colleagues helped to improve the content and presentation of this Letter. We wish to thank Lilia Anguelova, Stefano Cremonesi, Anatoly Dymarsky, Daniel Elander, Aki Hashimoto, Tim Hollowood, Prem Kumar, Dario Martelli, Ioannis Papadimitriou and Jorge Russo. The work of J.G. was funded by the DOE Grant DE-FG02-95ER40896. The work of E. C. and A. V. R. was funded in part by MICINN under grant FPA2008-01838, by the Spanish Consolider-Ingenio 2010 Programme CPAN (CSD2007-00042) and by Xunta de Galicia (Consellería de Educación and grant INCITE09 206 121 PR) and by FEDER. E. C. is supported by a Spanish FPU fellowship, and thanks the FRont Of Galician-speaking Scientists for unconditional support.

[1] S. Raby, S. Dimopoulos and L. Susskind, Nucl. Phys. B **169**, 373 (1980); H. Georgi, L. J. Hall and M. B. Wise, Phys. Lett. B **102**, 315 (1981) [Erratum-ibid. B **104**, 499 (1981)]; S. Dimopoulos, H. Georgi and S. Raby, Phys. Lett. B **127**, 101 (1983); G. Veneziano, Phys. Lett. B **102**, 139 (1981); E. Eichten and F. Feinberg, Phys. Lett. B **110**, 232 (1982).

[2] B. Holdom, Phys. Lett. B **150**, 301 (1985); K. Yamawaki *et al.* Phys. Rev. Lett. **56**, 1335 (1986); T. W. Appelquist *et al.* Phys. Rev. Lett. **57**, 957 (1986).

[3] N. Seiberg, Nucl. Phys. **B435**, 129-146 (1995). [hep-th/9411149].

[4] M. J. Strassler, [hep-th/0505153].

[5] A. Dymarsky, I. R. Klebanov, N. Seiberg, JHEP **0601**, 155 (2006). [hep-th/0511254].

[6] I. R. Klebanov and M. J. Strassler, JHEP **0008**, 052 (2000) [arXiv:hep-th/0007191].

[7] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [hep-th/9711200]. S. S. Gubser, I. R. Klebanov, A. M. Polyakov, Phys. Lett. **B428**, 105-114 (1998). [hep-th/9802109].E. Witten, Adv. Theor. Math. Phys. **2**, 253-291 (1998). [hep-th/9802150].

[8] N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz, Phys. Rev. D **58**, 046004 (1998) [hep-th/9802042].

[9] S. Weinberg, Phys. Rev. D **19**, 1277 (1979); L. Susskind, Phys. Rev. D **20**, 2619 (1979); S. Weinberg, Phys. Rev. D **13**, 974 (1976).

[10] S. Dimopoulos and L. Susskind, Nucl. Phys. B **155**, 237 (1979); E. Eichten and K. D. Lane, Phys. Lett. B **90**, 125 (1980).

[11] R. S. Chivukula, arXiv:hep-ph/0011264; K. Lane, arXiv:hep-ph/0202255, C. T. Hill and E. H. Simmons, Phys. Rept. **381**, 235 (2003) [Erratum-ibid. **390**, 553 (2004)] [arXiv:hep-ph/0203079]; A. Martin, arXiv:0812.1841 [hep-ph]; F. Sannino, arXiv:0911.0931 [hep-ph], M. Piai, Adv. High Energy Phys. **2010**, 464302 (2010). [arXiv:1004.0176 [hep-ph]].

[12] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998) [hep-th/9803131].

[13] J. M. Maldacena and C. Nunez, Phys. Rev. Lett. **86**, 588 (2001). [arXiv:hep-th/0008001].

[14] A. Butti, M. Grana, R. Minasian, M. Petrini and A. Zaffaroni, JHEP **0503**, 069 (2005) [arXiv:hep-th/0412187].

[15] E. Conde, J. Gaillard, C. Nunez, M. Piai and A. V. Ramallo, arXiv:1112.3350 [hep-th].

[16] G. Papadopoulos and A. A. Tseytlin, Class. Quant. Grav. **18**, 1333 (2001) [hep-th/0012034]; D. Cassani and A. F. Faedo, Nucl. Phys. B **843**, 455 (2011) [arXiv:1008.0883 [hep-th]]; I. Bena, G. Giecold, M. Grana, N. Halmagyi and F. Orsi, JHEP **1104**, 021 (2011) [arXiv:1008.0983 [hep-th]].

[17] A. H. Chamseddine and M. S. Volkov, Phys. Rev. Lett. **79**, 3343 (1997) [arXiv:hep-th/9707176].

[18] R. Casero, C. Nunez, A. Paredes, Phys. Rev. **D73**, 086005 (2006). [hep-th/0602027].

[19] R. Casero, C. Nunez, A. Paredes, Phys. Rev. **D77**, 046003 (2008). [arXiv:0709.3421 [hep-th]].

[20] C. Hoyos-Badajoz, C. Nunez, I. Papadimitriou, Phys. Rev. **D78**, 086005 (2008). [arXiv:0807.3039 [hep-th]].

[21] E. Conde, J. Gaillard, A. V. Ramallo, [arXiv:1107.3803 [hep-th]].

[22] A. Barranco, E. Pallante, J. G. Russo, JHEP **1109**, 086 (2011). [arXiv:1107.4002 [hep-th]].

[23] C. Nunez, A. Paredes and A. V. Ramallo, Adv. High Energy Phys. **2010**, 196714 (2010) [arXiv:1002.1088 [hep-th]].

[24] D. Elander, J. Gaillard, C. Nunez, M. Piai, JHEP **1107**, 056 (2011). [arXiv:1104.3963 [hep-th]].

[25] C. Nunez, I. Papadimitriou and M. Piai, Int. J. Mod. Phys. A **25**, 2837 (2010) [arXiv:0812.3655 [hep-th]].

[26] J. Maldacena and D. Martelli, JHEP **1001**, 104 (2010) [arXiv:0906.0591 [hep-th]].

[27] J. Gaillard, D. Martelli, C. Nunez and I. Papadimitriou, Nucl. Phys. B **843**, 1 (2011) [arXiv:1004.4638 [hep-th]].

[28] R. Minasian, M. Petrini, A. Zaffaroni, JHEP **1004**, 080 (2010). [arXiv:0907.5147 [hep-th]]; N. Halmagyi, [arXiv:1003.2121 [hep-th]]; E. Caceres, C. Nunez, L. A. Pando-Zayas, JHEP **1103**, 054 (2011). [arXiv:1101.4123 [hep-th]].S. Bennett, E. Caceres,

C. Nunez, D. Schofield and S. Young, arXiv:1111.1727 [hep-th].

[29] R. P. Andrews and N. Dorey, Nucl. Phys. B **751**, 304 (2006) [arXiv:hep-th/0601098]. R. P. Andrews and N. Dorey, Phys. Lett. B **631**, 74 (2005) [arXiv:hep-th/0505107].

[30] O. Aharony, JHEP **0103**, 012 (2001) [hep-th/0101013].